Example – Determining Repeatability and Reproducibility by Avova Method

The method of collecting the data is important in an ANOVA method. If the data are not collected in a random manner, this can lead to a source of bias values. A simple way to assure a balanced design for (*n*) parts, (*k*) appraisers, and (*r*) trials is through randomization. One common approach to randomization is to write *A1* on a slip of paper to denote the measurement for the first appraiser on the first part. Do this up to *A*(*n*) for the measurement by the first appraiser on the n_{th} part. Follow the same procedure for the next appraiser up to and including the k_{th} appraiser. The similar notation will be used where B_1, C_1 denotes the measurement for second and third appraiser on the first part. Once all *nk* combinations are written, then the slips of paper can be put in a hat or bowl. One at a time, a slip of paper is selected. These combinations (A_1, B_2, ...) are the measuring order in which the gage study will be performed. Once all *nk* combinations are selected, they are put back into the hat and the procedure is followed again. This is done for a total of *r* times to determine the order of experiments for each repeat. There are alternate approaches to generate a random sample. Care should be exercised to differentiate among random, haphazard and convenience sampling. In general, all efforts need to be taken to assure statistical independence within the study.

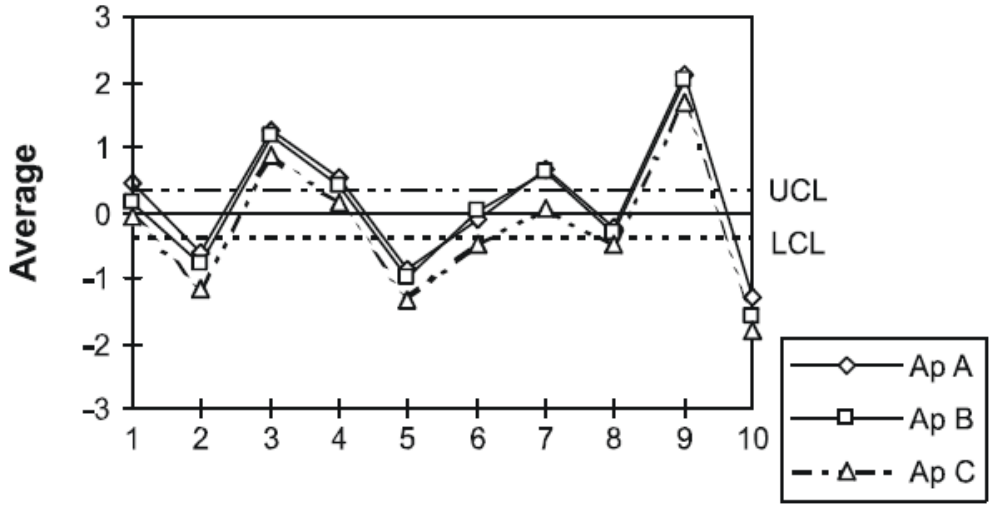
Using the same example as given in the Average and Range method. For our example, there are ten parts and three appraisers, and the experiment has been performed in random order three times for each part and appraiser combination.

Graphical Analysis:

Any of the graphical methods given in the discussion of Graphical Analysis in Average and Range can be used in the graphical analysis of the data collected as part of an ANOVA study.

Interaction plot:

This plot confirms the results of the *F* test on whether or not the interaction is significant. The points for each appraiser average measurement per part are connected to form *k* (number of appraisers) lines. The way to interpret the graph is if the *k* lines are parallel there is no interaction term. When the lines are nonparallel, the interaction can be significant. The larger the angle of intersection is, the greater is the interaction. Appropriate measures should be taken to eliminate the causes for the interaction. In this example the lines are nearly parallel, indicating no significant interaction.

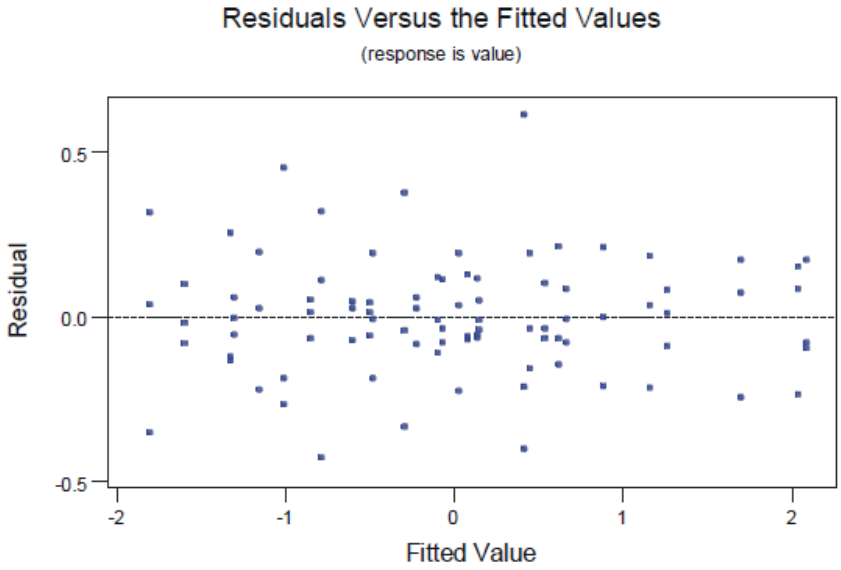


Interaction plot

Example – Determining Repeatability and Reproducibility by Avova Method

Residual plot:

This graph is more a check for the validity of the assumptions. This assumption is that the gage (error) is a random variable from a normal distribution. The residuals, which are the differences between the observed readings and predicted values, are plotted. Predicted value is the average of the repeated readings for each appraiser for each part. If the residuals are not randomly scattered above and below zero (horizontal reference line), it could be because the assumptions are incorrect and further investigation of the data is suggested.



Residual Plot

Anova table:

The ANOVA table here is composed of five columns .

- *Source* column is the cause of variation.
- *DF* column is the *degree of freedom* associated with the source.
- *SS* or *sum of squares* column is the deviation around the mean of the source.
- *MS* or *mean square* column is the sum of squares divided by degrees of freedom.
- *F-ratio* column, calculated to determine the statistical significance of the source value.

The ANOVA table is used to decompose the total variation into four components: parts, appraisers, interaction of appraisers and parts, and repeatability due to the instrument.
For analysis purposes, negative variance components are set to zero.
This information is used to determine the measurement system characteristics as in the Average and Range Method

Example – Determining Repeatability and Reproducibility by Avova Method

Source	DF	SS	MS	F
Appraiser	2	3.1673	1.58363	34.44*
Parts	9	88.3619	9.81799	213.52*
Appraiser by Part	18	0.3590	0.01994	0.434
Equipment	60	2.7589	0.04598	
Total	89	94.6471		

* Significant at $\alpha = 0.05$ level

Anova table

Estimate of Variance	Std. Dev. (σ)	% Total Variation	% Contribution
$\tau^2 = 0.039973$ (Repeatability)	$EV = 0.199933$	18.4	3.4
$\omega^2 = 0.051455$ (Appraiser)	$AV = 0.226838$	20.9	4.4
$\gamma^2 = 0$ (Interaction)	$INT = 0$	0	0
System = 0.09143 $(\tau^2 + \gamma^2 + \omega^2)$	$GRR = 0.302373$	27.9	7.8
$\sigma^2 = 1.086446$ (Part)	$PV = 1.042327$	96.0	92.2
Total Variation	$TV = 1.085$	100.0	

ANOVA Analysis % Variation & Contribution
(Estimate of variance is based on model without interaction)

Example – Determining Repeatability and Reproducibility by Avova Method

$$ndc = 1.41\left(1.04233 / .30237\right) = 4.861 \cong 5$$

$$Total\ Variation\ (TV) = \sqrt{GRR^2 + PV^2}$$

$$\%\ of\ TotalVariation = 100\left(\frac{\sigma_{(components)}}{\sigma_{(total)}}\right)$$

$$\%\ Contribution\ (to\ Total\ Variance) = 100\left(\frac{\sigma^2_{(components)}}{\sigma^2_{(total)}}\right)$$

Method	Lower 90% *CL*[55]	Std. Dev.	Upper 90% *CL*	% of Total Variation
*GRR**				
EV	0.175	.202	0.240	17.6
AV	0.133	.230	1.016	20.1
INTERACTION	--	na	--	na
GRR	0.266	.306	0.363	26.7
PV		1.104		96.4
ANOVA				
EV	0.177	0.200	0.231	18.4
AV	0.129	0.227	1.001	20.9
INTERACTION	--	0	--	0
GRR	0.237	0.302	1.033	27.9
PV		1.042		96.0

* In the average and range method, the interaction component cannot be estimated.

Comparison of ANOVA and Average and Range Methods

Example – Determining Repeatability and Reproducibility by Avova Method

Part No. & Name:	Gage Name:	Date:
Characteristics:	Gage No:	Performed by:
Specifications:	Gage Type:	

	STD. DEV.	% TOTAL VARIATION	PERCENT CONTRIBUTION
Repeatability (*EV*)	0.200	18.4	3.4
Reproducibility (*AV*)	0.227	20.9	4.4
Appraiser by Part (*INT*)	0	0	0
GRR	0.302	27.9	7.9
Part (*PV*)	1.042	96.0	92.2

Measurement System is acceptable for Process Control and Analysis.

Note:
Tolerance = N.A.
Total variation (*TV*) = 1.085
Number of distinct data categories (*ndc*) = 4

***GRR* ANOVA Method Report**