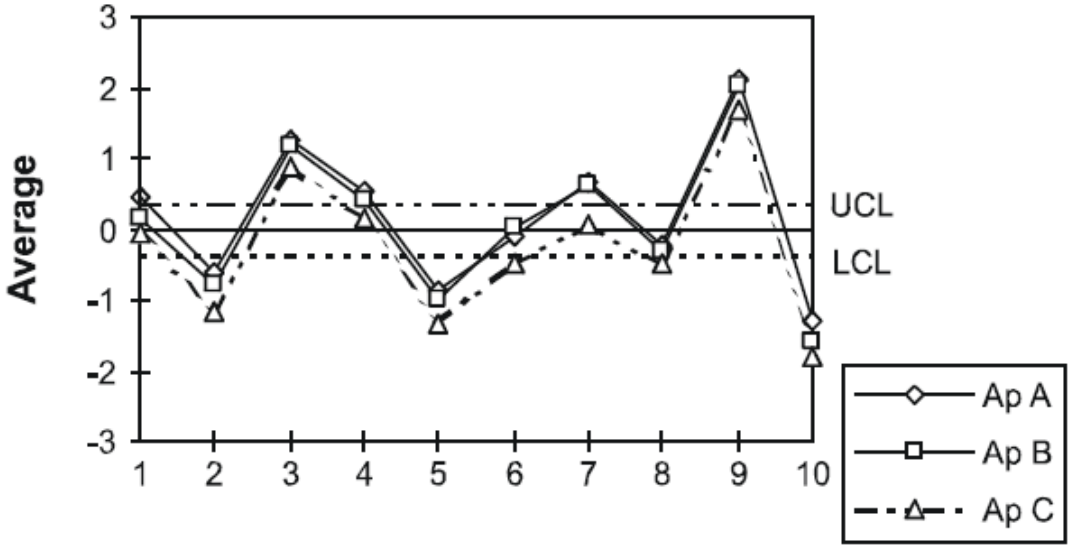
Example – Determining Repeatability and Reproducibility by Average and Range Method

Appraiser	Trial #	Part									
		1	2	3	4	5	6	7	8	9	10
A	1	0.29	-0.56	1.34	0.47	-0.80	0.02	0.59	-0.31	2.26	-1.36
	2	0.41	-0.68	1.17	0.50	-0.92	-0.11	0.75	-0.20	1.99	-1.25
	3	0.64	-0.58	1.27	0.64	-0.84	-0.21	0.66	-0.17	2.01	-1.31
B	1	0.08	-0.47	1.19	0.01	-0.56	-0.20	0.47	-0.63	1.80	-1.68
	2	0.25	-1.22	0.94	1.03	-1.20	0.22	0.55	0.08	2.12	-1.62
	3	0.07	-0.68	1.34	0.20	-1.28	0.06	0.83	-0.34	2.19	-1.50
C	1	0.04	-1.38	0.88	0.14	-1.46	-0.29	0.02	-0.46	1.77	-1.49
	2	-0.11	-1.13	1.09	0.20	-1.07	-0.67	0.01	-0.56	1.45	-1.77
	3	-0.15	-0.96	0.67	0.11	-1.45	-0.49	0.21	-0.49	1.87	-2.16

The use of graphical tools is very important. The specific graphical tools used depend on the experimental design employed to collect the data. A systematic screening of the data for apparent special causes of variations by using graphical tools should precede any other statistical analysis. The following are some of the techniques which have proven to be useful.

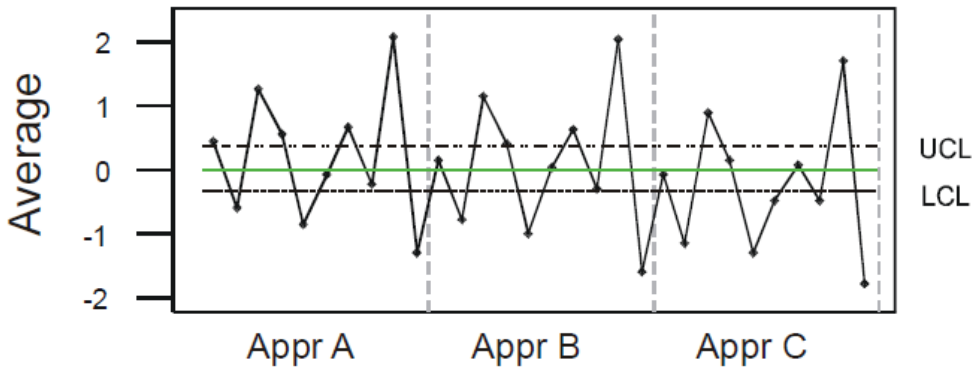
Average Chart:

The averages of the multiple readings by each appraiser on each part are plotted by appraiser with part number as an index. This can assist in determining consistency between appraisers. The grand average and control limits determined by using the average range are also plotted. The resulting Average Chart provides an indication of "usability" of the measurement system. The area within the control limits represents the measurement sensitivity ("noise"). Since the group of parts used in the study represents the process variation, approximately one half or more of the averages should fall outside the control limits. If the data show this pattern, then the measurement system should be adequate to detect part-to-part variation and the measurement system can provide useful information for analyzing and controlling the process. If less than half fall outside the control limits then either the measurement system lacks adequate effective resolution or the sample does not represent the expected process variation.



Average Chart – "Stacked"

Example – Determining Repeatability and Reproducibility by Average and Range Method



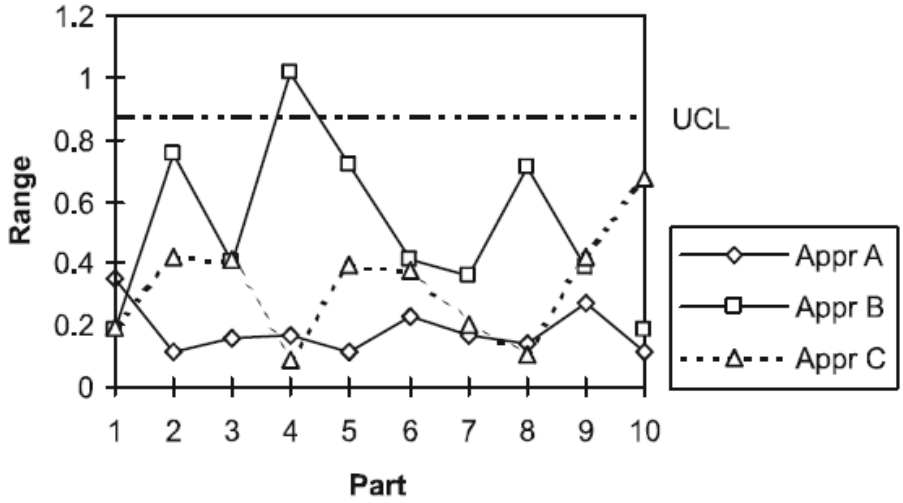
Average Chart – "Unstacked"

Review of the charts indicates that the measurement system appears to have sufficient discrimination for processes with variation described by the sample parts. No appraiser-to-appraiser differences are readily apparent.

Range Chart:

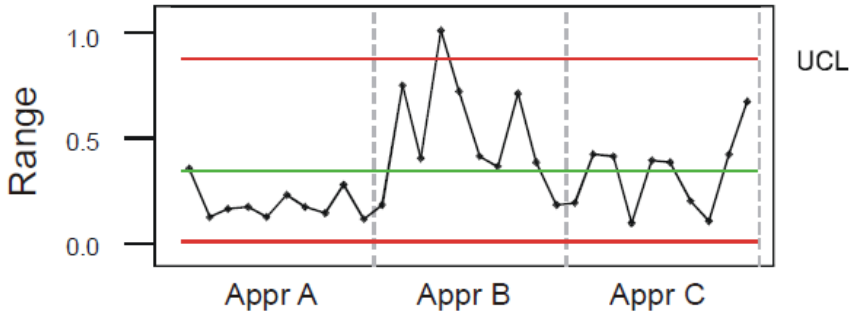
The range control chart is used to determine whether the process is in control. The reason being that no matter how large the measurement error may be, the control limits will allow for that error. That is why the special causes need to be identified and removed before a measurement study can be relevant. The ranges of the multiple readings by each appraiser on each part are plotted on a standard range chart including the average range and control limit(s). From the analysis of the data that are being plotted, several useful interpretations can be made. If all ranges are in control, all appraisers are doing the same job. If one appraiser is out-of-control, the method used differs from the others. If all appraisers have some out of control ranges, the measurement system is sensitive to appraiser technique and needs improvement to obtain useful data. The range chart can assist in determining:

- Statistical control with respect to repeatability
- Consistency of the measurement process between appraisers for each part



Range Chart – "Stacked"

Example – Determining Repeatability and Reproducibility by Average and Range Method



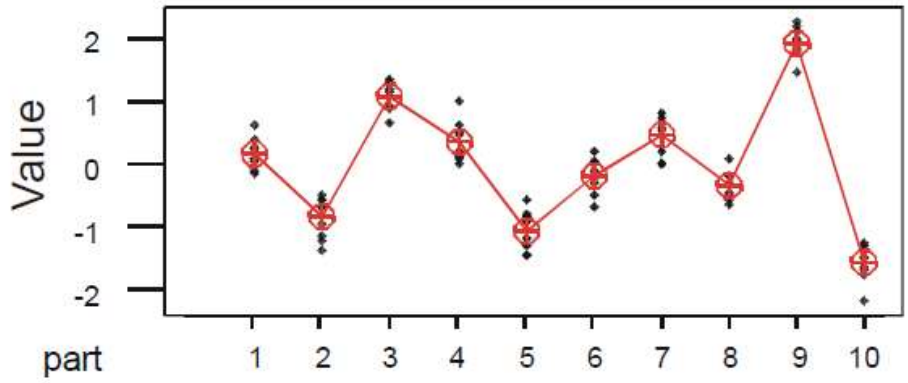
Range Chart – "Unstacked"

Review of the above charts indicates that there are differences between the variability of the appraisers.

Run Chart:

The individual readings are plotted by part for all appraisers to gain insight into:

- The effect of individual parts on variation consistency
- Indication of outlier readings (i.e., abnormal readings)



Run Chart by Part

Review of the above chart does not indicate any outliers or inconsistent parts.

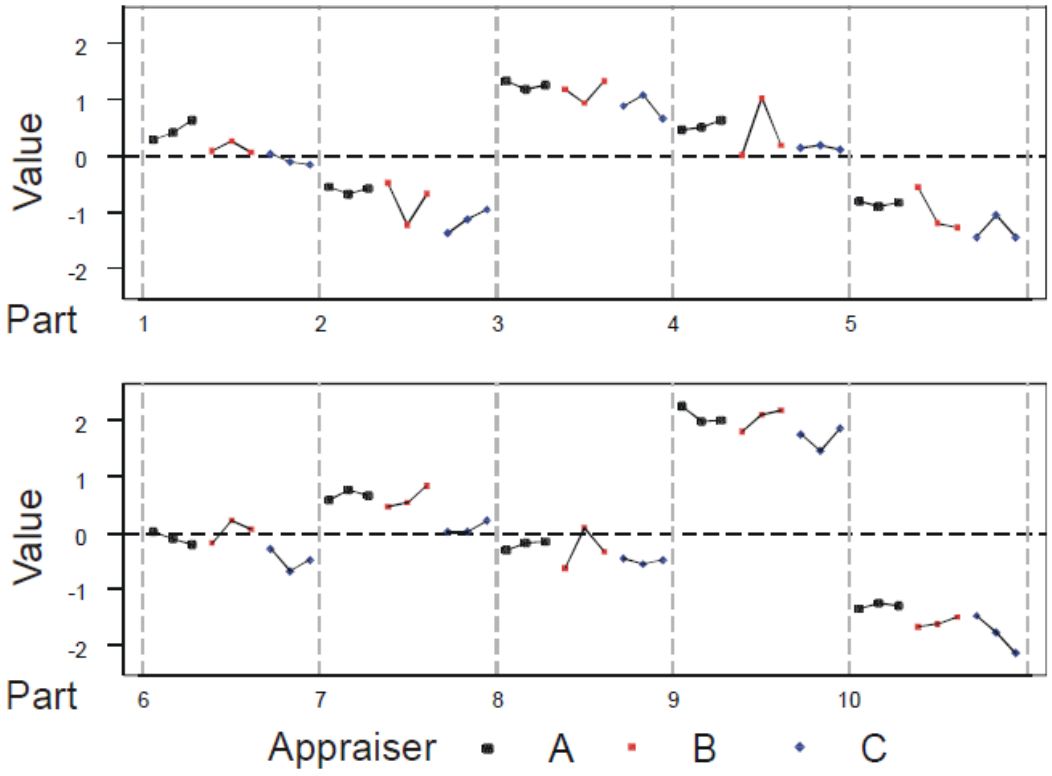
Scatter Plot:

The individual readings are plotted by part-by-appraiser to gain insight into:

- Consistency between appraisers
- Indication of possible outliers
- Part-appraiser interactions

Review of the Figure does not indicate any significant outliers but does indicate that appraiser C may have lower readings than the other appraisers.

Example – Determining Repeatability and Reproducibility by Average and Range Method



Scatter Plot

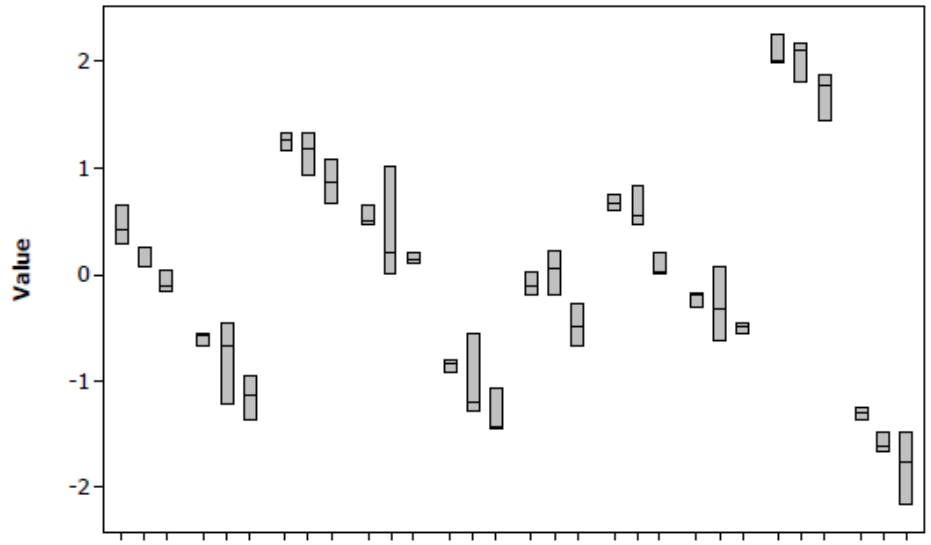
Whiskers Chart:

In a Whiskers Chart, the high and low data values and the average by partby-appraiser are plotted . This provides insight into:

- Consistency between appraisers
- Indication of outliers
- Part-appraiser interactions

Review of Figure does not indicate any significant outliers but indicates that appraiser B may have the most variability.

Example – Determining Repeatability and Reproducibility by Average and Range Method



Whiskers Chart

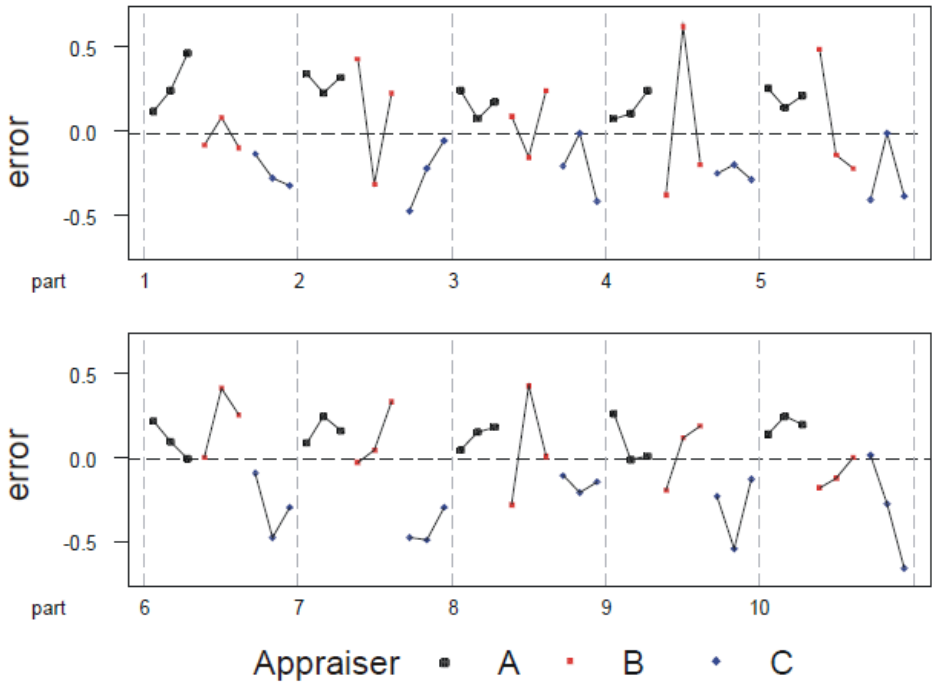
Error chart:

The data from the measurement system analysis can be analyzed by running "Error Charts" of the individual deviations from the accepted reference values. The individual deviation or error for each part is calculated as follows:

Error = Observed Value – Reference Value or

Error = Observed Value – Average Measurement of the Part

This depends upon whether or not reference values of the data being measured are available.



Error Charts

Example – Determining Repeatability and Reproducibility by Average and Range Method

Review of the above charts indicates:

- Appraiser A has an overall positive bias
- Appraiser B has the most variability but no apparent bias.
- Appraiser C has an overall negative bias

Normalized Histogram:

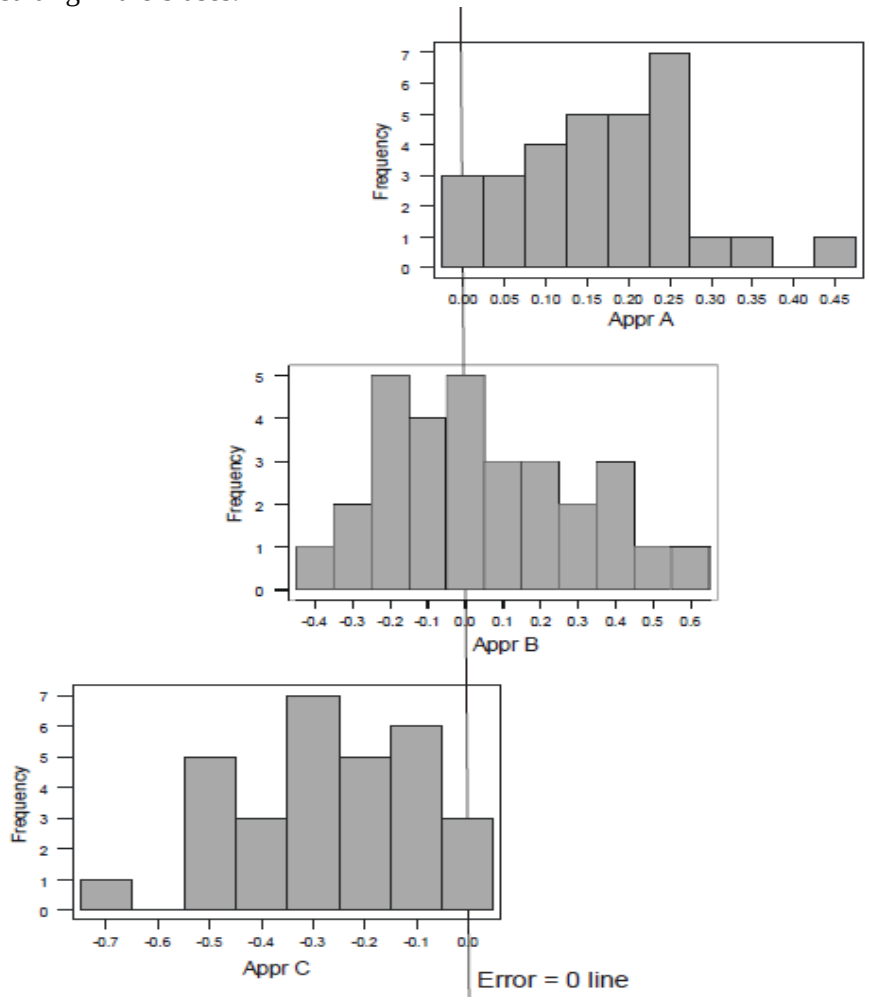
The histogram plot is a graph that displays the frequency distribution of the gage error of appraisers who participated in the study. It also shows their combined frequency distribution. If the reference values are available:

$$Error = Observed\ Value - Reference\ Value$$

Otherwise:

$$Normalized\ Value = Observed\ Value - Part\ Average$$

The histogram plot provides a quick visual overview of how the error is distributed. Issues such as whether bias or lack of consistency exists in the measurements taken by the appraisers can be identified even before the data are analyzed. Analysis of the histograms reinforces that of the error charts. They also indicate that only appraiser B has a symmetric form. This may indicate that appraisers A and C are introducing a systematic source of variation which is resulting in the biases.



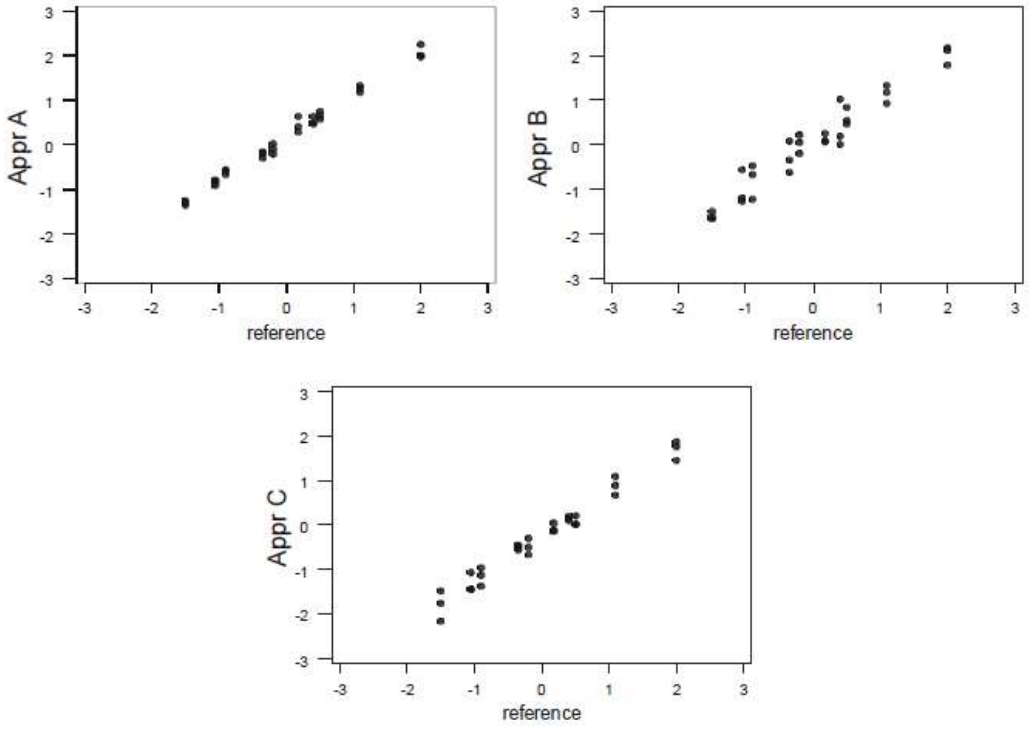
Normalized Histogram

Example – Determining Repeatability and Reproducibility by Average and Range Method

X-Y Plot of Averages by Size:

The averages of the multiple readings by each appraiser on each part are plotted with the reference value or overall part averages as the index. This plot can assist in determining:

- Linearity (if the reference value is used)
- Consistency in linearity between appraisers

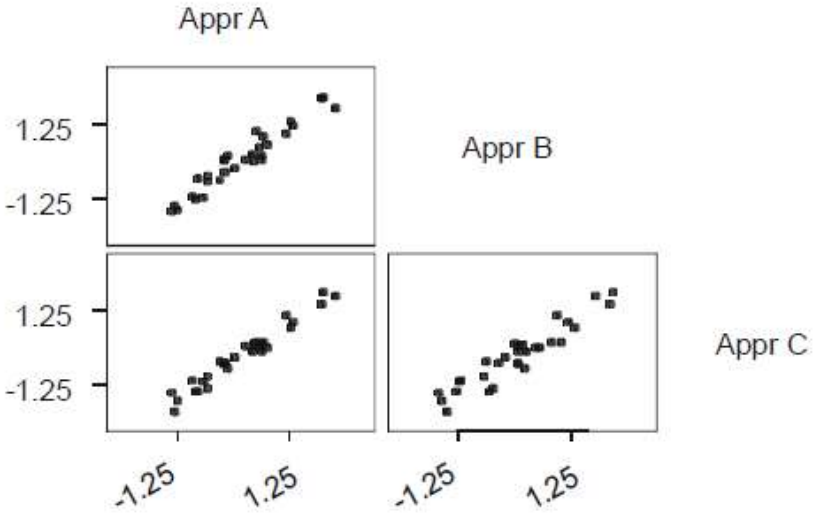


X–Y Plot of Averages by Size

Comparison X-Y Plots

The averages of the multiple readings by each appraiser on each part are plotted against each other with the appraisers as indices. This plot compares the values obtained by one appraiser to those of another . If there were perfect agreement between appraisers, the plotted points would describe a straight line through the origin and 45° to the axis.

Example – Determining Repeatability and Reproducibility by Average and Range Method



Comparison X–Y Plots

Example – Determining Repeatability and Reproducibility by Average and Range Method

Gage Repeatability and Reproducibility Data Collection Sheet

	Appraiser /Trial #	PART 1	2	3	4	5	6	7	8	9	10	AVERAGE
1	A 1	0.29	-0.56	1.34	0.47	-0.80	0.02	0.59	-0.31	2.26	-1.36	0.194
2	2	0.41	-0.68	1.17	0.50	-0.92	-0.11	0.75	-0.20	1.99	-1.25	0.166
3	3	0.64	-0.58	1.27	0.64	-0.84	-0.21	0.66	-0.17	2.01	-1.31	0.211
4	Average	**0.447**	**-0.607**	**1.260**	**0.537**	**-0.853**	**-0.100**	**0.667**	**-0.227**	**2.087**	**-1.307**	$\bar{X}_a$ = **0.1903**
5	Range	**0.35**	**0.12**	**0.17**	**0.17**	**0.12**	**0.23**	**0.16**	**0.14**	**0.27**	**0.11**	$\bar{R}_a$ = **0.184**
6	B 1	0.08	-0.47	1.19	0.01	-0.56	-0.20	0.47	-0.63	1.80	-1.68	0.001
7	2	0.25	-1.22	0.94	1.03	-1.20	0.22	0.55	0.08	2.12	-1.62	0.115
8	3	0.07	-0.68	1.34	0.20	-1.28	0.06	0.83	-0.34	2.19	-1.50	0.089
9	Average	**0.133**	**-0.790**	**1.157**	**0.413**	**-1.013**	**0.027**	**0.617**	**-0.297**	**2.037**	**-1.600**	$\bar{X}_b$ = **0.0683**
10	Range	**0.18**	**0.75**	**0.40**	**1.02**	**0.72**	**0.42**	**0.36**	**0.71**	**0.39**	**0.18**	$\bar{R}_b$ = **0.513**
11	C 1	0.04	-1.38	0.88	0.14	-1.46	-0.29	0.02	-0.46	1.77	-1.49	-0.223
12	2	-0.11	-1.13	1.09	0.20	-1.07	-0.67	0.01	-0.56	1.45	-1.77	-0.256
13	3	-0.15	-0.96	0.67	0.11	-1.45	-0.49	0.21	-0.49	1.87	-2.16	-0.284
14	Average	**-0.073**	**-1.157**	**0.880**	**0.150**	**-1.327**	**-0.483**	**0.080**	**-0.503**	**1.697**	**-1.807**	$\bar{X}_c$ = **-0.2543**
15	Range	**0.19**	**0.42**	**0.42**	**0.09**	**0.39**	**0.38**	**0.20**	**0.10**	**0.42**	**0.67**	$\bar{R}_c$ = **0.328**
16	Part Average	**0.169**	**-0.851**	**1.099**	**0.367**	**-1.064**	**-0.186**	**0.454**	**-0.342**	**1.940**	**-1.571**	$\bar{\bar{X}}$ = **.0014** R_p = **3.511**
17	([$\bar{R}_a$ = **0.184**] + [$\bar{R}_b$ = **0.513**] + [$\bar{R}_c$ = **0.328**]) / [# OF APPRAISERS = **3**] = **0.3417**											$\bar{\bar{R}}$ = **0.3417**
18	[Max $\bar{X}$ = **0.1903**] – [Min $\bar{X}$ = **-0.2543**] = $\bar{X}_{DIFF}$ = **0.4446**											
19	* [$\bar{\bar{R}}$ = **0.3417**] x [D_4 = **2.58**] = UCL_R = **0.8816**											

*D_4 = 3.27 for 2 trials and 2.58 for 3 trials. UCL_R represents the limit of individual R's. Circle those that are beyond this limit. Identify the cause and correct. Repeat these readings using the same appraiser and unit as originally used or discard values and re-average and recompute $\bar{\bar{R}}$ and the limiting value from the remaining observations.

Notes:_____

Example – Determining Repeatability and Reproducibility by Average and Range Method

Gage Repeatability and Reproducibility Report

Part No. & Name: | Gage Name: | Date:
Characteristics: | Gage No: | Performed by:
Specifications: | Gage Type:

From data sheet: $\bar{\bar{R}}$ = **0.3417** $\bar{X}_{DIFF}$ = **0.4446** R_p = **3.511**

Measurement Unit Analysis	% Total Variation (*TV*)
Repeatability – Equipment Variation (*EV*) $EV = \bar{\bar{R}} \times K_1$ = **0.3417** × **0.5908** = **0.20188**	$\%EV = 100\ [EV/TV]$ = 100 [**0.20188**/**1.14610**] = **17.62%**
Reproducibility – Appraiser Variation (*AV*) $AV = \sqrt{(\bar{X}_{DIFF} \times K_2)^2 - (EV^2/(nr))}$ $= \sqrt{(0.4446 \times 0.5231)^2 - (0.20188^2/(10 \times 3))}$ = **0.22963** *n* = *parts* *r* = *trials*	$\%AV = 100\ [AV/TV]$ = 100 [**0.22963**/**1.14610**] = **20.04%**
Repeatability & Reproducibility (*GRR*) $GRR = \sqrt{EV^2 + AV^2}$ $= \sqrt{(0.20188^2 + 0.22963^2)}$ = **0.30575**	$\%GRR = 100\ [GRR/TV]$ = 100 [= **0.30575**/**1.14610**] = **26.68%**
Part Variation (*PV*) $PV = R_p \times K_3$ = **1.10456**	$\%PV = 100\ [PV/TV]$ = 100 [**1.10456**/**1.14610**] = **96.38%**
Total Variation (*TV*) $TV = \sqrt{GRR^2 + PV^2}$ $= \sqrt{(0.30575^2 + 1.10456^2)}$ = **1.14610**	$ndc = 1.41\left(PV/GRR\right)$ = 1.41(**1.10456**/**0.30575**) = **5.094 ~ 5**

Trials	K_1
2	0.8862
3	0.5908

Appraisers	2	3
K_2	0.7071	0.5231

Parts	K_3
2	0.7071
3	0.5231
4	0.4467
5	0.4030
6	0.3742
7	0.3534
8	0.3375
9	0.3249
10	0.3146

Gage Repeatability and Reproducibility Report

Example – Determining Repeatability and Reproducibility by Average and Range Method