Example – Determining Linearity

A plant supervisor was introducing a new measurement system to the process. Five parts were chosen throughout the operating range of the measurement system based upon documented process variation. Each part was measured by layout inspection to determine its reference value. Each part was then measured twelve times by the lead operator. The parts were selected at random during the study.

	Part	1	2	3	4	5
	Reference Value	2.00	4.00	6.00	8.00	10.00
	1	2.70	5.10	5.80	7.60	9.10
	2	2.50	3.90	5.70	7.70	9.30
	3	2.40	4.20	5.90	7.80	9.50
T	4	2.50	5.00	5.90	7.70	9.30
R	5	2.70	3.80	6.00	7.80	9.40
I	6	2.30	3.90	6.10	7.80	9.50
A	7	2.50	3.90	6.00	7.80	9.50
L	8	2.50	3.90	6.10	7.70	9.50
S	9	2.40	3.90	6.40	7.80	9.60
	10	2.40	4.00	6.30	7.50	9.20
	11	2.60	4.10	6.00	7.60	9.30
	12	2.40	3.80	6.10	7.70	9.40

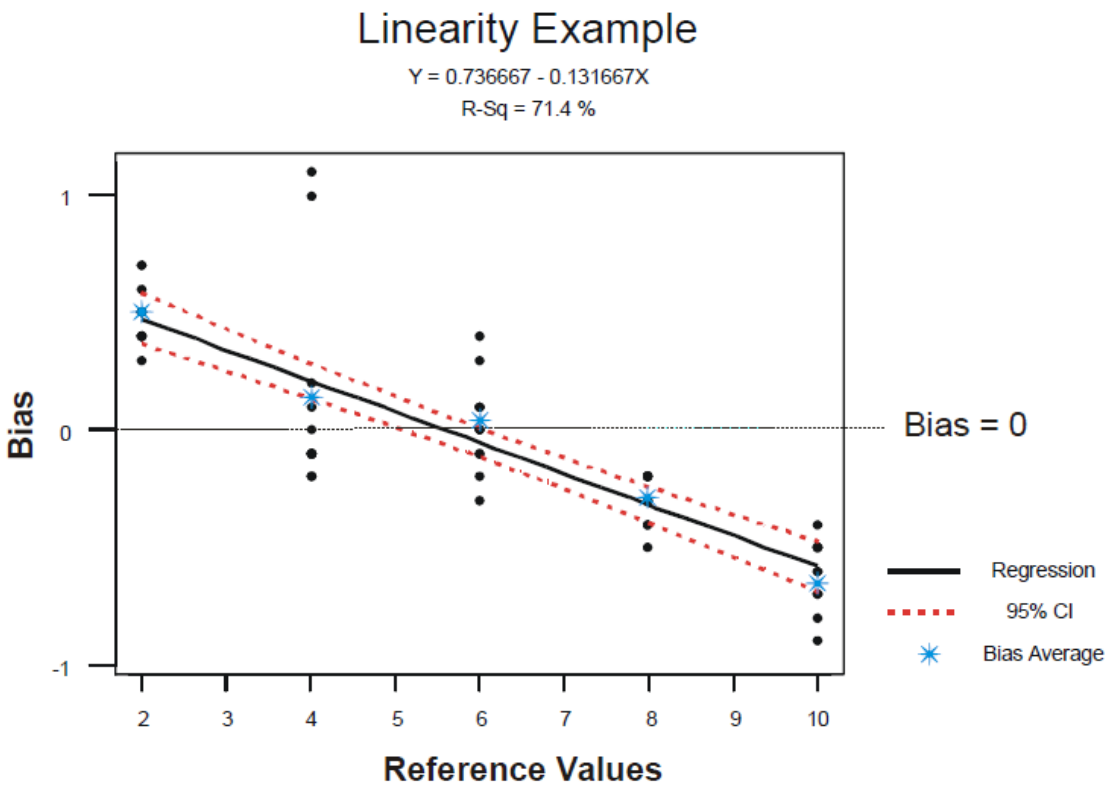
Linearity Study Data

Using a spreadsheet and statistical software, the supervisor generated then linearity plot

	Part	1	2	3	4	5
	Reference Value	2.00	4.00	6.00	8.00	10.00
	1	0.7	1.1	-0.2	-0.4	-0.9
	2	0.5	-0.1	-0.3	-0.3	-0.7
	3	0.4	0.2	-0.1	-0.2	-0.5
B	4	0.5	1.0	-0.1	-0.3	-0.7
I	5	0.7	-0.2	0.0	-0.2	-0.6
A	6	0.3	-0.1	0.1	-0.2	-0.5
S	7	0.5	-0.1	0.0	-0.2	-0.5
	8	0.5	-0.1	0.1	-0.3	-0.5
	9	0.4	-0.1	0.4	-0.2	-0.4
	10	0.4	0.0	0.3	-0.5	-0.8
	11	0.6	0.1	0.0	-0.4	-0.7
	12	0.4	-0.2	0.1	-0.3	-0.6
	BIAS Avg.	0.491667	0.125	0.025	-0.29167	-0.61667

Intermediate Results

Example – Determining Linearity



Graphical Analysis

The graphical analysis indicates that special causes may be influencing the measurements system. The data for reference value 4 appear to be bimodal. Even if the data for reference value 4 were not considered, the graphical analysis clearly shows that this measurement system has a linearity problem. The R^2 value indicates that a linear model may not be an appropriate model for these data. F Even if the linear model is accepted, the "*bias = 0*" line intersects the confidence bounds rather than being contained by them. At this point, the supervisor ought to begin problem analysis and resolution on the measurement system, since the numerical analysis will not provide any additional insights. However, wanting to make sure no paperwork is left unmarked, the supervisor calculates the *t*-statistic for the slope and intercept:

$$t_a = -12.043$$

$$t_b = 10.158$$

Taking the default $\alpha = .05$ and going to the *t*-tables with $(gm - 2) = 58$ degrees of freedom and a proportion of .975, the supervisor comes up with the critical value of:

$$t_{58,.975} = 2.00172$$

Since $|t_a| > t_{58,.975}$, the result obtained from the graphical analysis is reinforced by the numerical analysis – there is a linearity problem with this measurement system.

If the measurement system has a linearity problem, it needs to be recalibrated to achieve zero bias through the modification of the hardware, software or both. If the bias cannot be adjusted to zero bias throughout the measurement system range, it still can be used for product/process control but not analysis as long as the measurement system remains stable.